

GCSE MARKING SCHEME

AUTUMN 2023

GCSE
MATHEMATICS
UNIT 2 – HIGHER TIER
3300U60-1

INTRODUCTION

This marking scheme was used by WJEC for the 2023 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

WJEC GCSE MATHEMATICS

AUTUMN 2023 MARKING SCHEME

GCSE Mathematics Unit 2: Higher Tier	Mark	Comments
	IVICITY	Commente
1.(a) 1 10	B2	B1 for each. Table takes precedence if conflicting values given.
1.(b) At least 4 correct plots and no incorrect plot.	P1	FT 'their (-2,1)' and 'their (1,10)' OR (-2,1) and (1,10) plotted. Allow ± '½ a small square'.
A smooth <u>curve</u> drawn through their plots.	C1	FT 'their 6 plots'. OR a curve through the 4 given points AND (-2,1) and (1,10) Allow intention to pass through their plots. (± 1 small square horizontally or vertically.)
2. (a) (Area =) 7.4 + 9.1 × 5.7 or equivalent 2	M1	(= 47·025) May be seen in stages. Allow M1 for correct intent seen. e.g. 7·4 + 9·1 × 5·7 ÷ 2
× 15-6	m1	
Allow an answer from 733 to 734 (cm ³) inclusive.	A1	CAO Note: 733-59 or 733-6 (cm³)
2. (b) 733-59 × 19-3	M1	FT 'their volume from (a)' × 19·3
14158(·287(g))	A1	
Allow an answer from 14·1 to 14·2 (kg) inclusive.	B1	FT 'their 14158·287' ÷ 1000 Allow 14 from correct working. Note: 14·158(287) or 14·16 or 14·2 (kg)
2.(b) Alternative method (converting to g first)		
0·0193 (kg/cm³) 733·59 × 0·0193	B1 M1	FT 'their volume from (a)' and FT 'their 0.0193' provided a place value error has been made.
Allow an answer from 14-1 to 14-2 (kg) inclusive	A1	Allow 14 from correct working. Note: 14·158(287) or 14·16 or 14·2 (kg)

3. Identifying or implying that there are 16 possible correct combinations (e.g 2 × 6) or products (e.g.12)	B1	Award B1 for • simply stating 16 • (4 × 4 =)16 • completed sample space (need not be correct) • sight of $\frac{1}{4} \times \frac{1}{4}$ • sight of 16 in a denominator.
Identifies <u>all</u> possible combinations (e.g 2 × 6) or products (e.g 12) that are a factor of 36 $1 \times 6 = 6$, $1 \times 9 = 9$, $2 \times 6 = 12$	B2	FT 'their 16 possible correct products'. If products not used (e.g 2 + 6 = 8), do not award B2 or B1.
$2 \times 9 = 18$, $3 \times 6 = 18$, $4 \times 9 = 36$		 Award B2 for clearly identifying one of the following: the 6 (and no more) combinations 1 × 6, 2 × 9, etc that form factors of 36 that can be achieved by the two spinners the 6 (and no more) products of factors of 36 that can be achieved by the two spinners: 6, 9, 12, 18, 18, 36 sight of 6 × ½ × ½ or equivalent.
		 Award B1 for clearly identifying one of the following: at least 4 combinations that are factors of 36 at least 4 products of factors of 36 that can be achieved by the two spinners: 6, 9, 12, 18, 36 all of the factors of 36 (1,2,3,4,6,9,12,18,36).
(Probability factor of 36 =) 6 or equivalent. ISW 16	B1	FT 'their list' only if at least 12 combinations or products given with at least two factors of 36 that can be achieved by the two spinners clearly identified.
		Penalise, −1, any incorrect notation e.g. '6 out of 16'.
		Unsupported 6 or 3 or equivalent gains B1 B2 B1.
Organisation and Communication	OC1	For OC1, candidates will be expected to: • present their response in a structured way • explain to the reader what they are doing at each step of their response • lay out their explanation and working in a way that is clear and logical • write a conclusion that draws together their results and explains what their answer means
Accuracy of writing	W1	For W1, candidates will be expected to:

4. $(AC^2 =) 8^2 + 4.5^2 \text{ or equivalent}$	M1	Check diagram. note: $(AC^2 =) 64 + 20.25$.
(AC =) 6 + 4.5 or equivalent	IVII	
$(AC =) \sqrt{8^2 + 4.5^2}$ or equivalent	m1	note: $(AC =) \sqrt{84.25}$. FT $\sqrt{100}$ for m1 only provided M1 gained.
9-1(7878) (cm) ISW	A1	Accept the answer rounded or truncated to at least one decimal place.
		Final answer of • AC = 84.25 is M1m0A0.
4. Alternative method to find AC using Trig A correct and complete method (using trigonometric relationships)	M2	
9·1(7878) (cm) ISW	A1	Accept the answer rounded or truncated to at least one decimal place.
5. One correct evaluation $3 \le x \le 4$ 2 correct evaluations $3.75 \le x \le 3.95$, (one value < 80, one value > 80)	B1 B1	Correct evaluation regarded as enough to identify if $< 80 \text{ or } > 80$. If evaluations not seen accept 'too high' or 'too low'. Look out for $x^3 + 6x - 80 = 0$
2 correct evaluations $3.75 \le x \le 3.85$, (one value < 80, one value > 80)	M1	$\frac{x}{3}$ $\frac{x^3 + 6x}{45}$
x = 3.8	A1	3.1 48·391 3.2 51·968 3.3 55·737 3.4 59·704 3.5 63·875
		3.5 63·875 3.6 68·256 3.75 75·2343
		3.7 72.853 3.84 79.6631
		3.8 77·672 3.85 80·1666
		3.9 82·719 3.95 85·3298 4 88
		Unsupported $x = 3.8$ is awarded B0B0M0A0. An answer of $x = 3.8$ can only be awarded M1A1, following sight of 2 correct evaluations $3.75 \le x \le 3.85$ (one evaluation < 80, one evaluation > 80). If 3.85 is given as 80 (truncated) award M0 A0 unless 'too high' or equivalent is indicated.

6. Sight of $(5x + 3)(2x - 1)$ or $2(5x + 3)(2x - 1)$ or equivalent	S1	Intention to (2 x) width x length. Allow 4 x width x length or equivalent for S1. May be implied in later working if B2 or B1 awarded.
(total area of both rectangles = $20x^2 + 2x - 6$ (cm ²)	B2	Mark final answer for B2. Allow $20x^2 + 2x + -6$ for B2.
		Award B1 for sight of one of the following: • $20x^2 + 12x - 10x - 6$ with at least three terms out of the four correct (must have x^2 term) • $10x^2 + 6x - 5x - 3$ • $2(10x^2 + x - 3)$ • $10x^2 + x - 3$ • $40x^2 + 4x - 12$. If no marks, award SC1 for one of the following: • $40x^2 + 24x - 20x - 12$. • $20x^2 + 22x + 6$ from $2(5x + 3)(2x + 1)$ • $20x^2 - 2x - 6$ from $2(5x - 3)(2x + 1)$ • $20x^2 - 22x + 6$ from $2(5x - 3)(2x - 1)$.
7.		Check diagram for answer.
Method using angle XYZ $YZ = 18.6 \text{or} 18.6 \times \sin 50 \text{or equivalent}$ $\tan 40(^{\circ}) \sin 40$	M2	Award M1 for one of the following • $tan 40(°) = \underline{18.6}$ YZ • $\underline{YZ} = \underline{18.6}$ or equivalent $\underline{sin 50}$ $\underline{sin 40}$
= 22(·166)(cm)	A1	Accept an answer rounded or truncated.
		Award M2A0 for any of the following unsupported answers: Method Radians Gradians
		18·6 tan 40 -16·648 25·600
7. Alternative using angle YXZ YZ = 18·6 × tan 50(°)	M2	Award M1 for tan $50(^{\circ}) = \underline{YZ}$ 18.6
= 22(·166)(cm)	A1	Accept an answer rounded or truncated
		Award M2A0 for any of the following unsupported answers:
		Method Radians Gradians
		18·6 × tan 50 -5·057 18·6
7. <u>Alternative method</u> Correct use of a 'two-step' method.	M2	A partial trigonometric method is M0.
22(·166)(cm) ISW	A1	Accept an answer rounded or truncated.

8.			
Working in mm 60.5×7 \mathbf{OR} $420 + 0.5 \times 7$	Working in cm 6.05×7 OR $42 + 0.05 \times 7$	M1	Allow $60 <$ 'their 60.5 ' ≤ 61 . Allow $6 \text{ cm} <$ 'their 6.05 ' $\text{cm} \leq 6.1 \text{ cm}$.
	423-5 (mm) ISW	A1	Allow 42·35 cm, provided units are given and correct. CAO.
			If no marks, award SC1 for sight of 60⋅5 OR 6⋅05.
9. Midpoints 25, 35, 45, (5	5), 65, 75	B1	May be implied in later working (i.e the correct products).
ı	Missing 10 for $50 \le t < 60$	B1	producto).
25×2 + 35×8 + 45×4 + (= 50 + 280 + 180 + 550 +		M1	FT 'their 10' provided \neq 0 or 1. Allow with consistent incorrect midpoints provided at least 5 within the correct interval including 'bounds' Allow use of a instead of 10 (sight of 1080 + 55a).
	÷ 32	m1	FT 22 + 'their a ' (a \neq 0). Allow use of a instead of 10.
50.9(375)	or 51 or equivalent ISW	A1	CAO. Must be derived from correct working.
			If no marks or first B1 only, award SC1 for one of the following: • $(1080 \div 22 =) 49(\cdot 09)$ from use of $a = 0$ • $(1080 \div 32 =) 33.7(5)$ or 34 from use of $a = 0$ • $(1135 \div 23 =) 49(\cdot 3)$ from use of $a = 1$ • $(1135 \div 32 =) 35(\cdot 46875)$ from use of $a = 1$. Award B1 B0 M1 m1 A0 for $\frac{1080 + 55a}{22 + a}$ or $\frac{1080 + 55a}{32}$ or equivalent expression involving a .
10. Sight of $12x + 4y = 180$ or AND $26x + 6$	equivalent $-7y = 360$ or equivalent	B2	x and y terms need to be collected for B2. If B2 not awarded, award B1 for one of the following: • $12x + 4y = 180$ or equivalent • $13x + 5x + 8x + 7y = 360$ • $26x + 7y = 360$ or equivalent
Method to eliminate one va e.g. equal coefficients AND add or subtract or use a me	appropriate intention to	M1	FT 'their equations', provided of equivalent difficulty. Allow one error in one term (not the term with equal coefficients).
First variable found x	= 9(°) or $y = 18(°)$	A1	CAO (for their equations).
Substitute to find the 2 nd variable.		m1	FT substitution of their '1 st variable' if M1 gained.
Second variable found.		A1	No marks for 'trial and improvement'. No marks for an unsupported answer.

11.(a) $(2x+5)(3x+2)$ ISW	B2	B1 for one of the following:
		 (2x 5)(3x 2) A pair of brackets which leads to the
		expansion of $6x^2 + bx + c$ where either
		b = 19 or c = 10
		• $\frac{(2ax+5a)(3bx+2b)}{ab}$ [e.g. $(x+2.5)(6x+4)$]`
11.(b) $m(m+5)(m-5)$	В3	Mark final answer for B3.
		B2 for one of the following: • $(m+5)(m^2-5m)$
		• $(m+3)(m-3m)$ • $(m-5)(m^2+5m)$
		• $m(m5)(m5)$
		$\bullet (m+5)(m-5)$
		B1 for one of the following:
		• sight of $m(m^2 - 25)$
		• sight of $(m + 5)(m - 5)$ included within an expression, e.g. $m^2(m + 5)(m - 5)$
		• $(m 5)(m 5)$
		, , , ,
11.(c) $(p+7)(p+31)$ ISW	B2	B1 for $(p + 7)(p + 29 + 2)$ OR Allow B1 for $(p + 7)(p + k)$ with $k \neq 0$ or 2 or 7 or 29.
Alternative method		7.110W B 1 101 (\$ 1 7)(\$ 1 10) WILLIN 7 0 01 2 01 7 01 20.
$(p+7)(p+29) + 2(p+7) = p^2 + 38p + 217$	-	No mark for the expansion and collection of terms.
= (p+7)(p+31) ISW	B2	Award B1 for a correct factorisation, if possible, on FT from 'their derived quadratic expression', provided no
		more than one error.
		SC1 for an answer of $(p+7)(p+k)$ with $k \neq 0$ or 7
12. $(BC^2 =) 36 \cdot 1^2 + 13 \cdot 8^2 - 2 \times 36 \cdot 1 \times 13 \cdot 8 \times \cos 29 \text{ OR}$	M1	or 2 or 29.
(BC =) $\sqrt{[36 \cdot 1^2 + 13 \cdot 8^2 - 2 \times 36 \cdot 1 \times 13 \cdot 8 \times \cos 29]}$	IVII	
(BC =) 24·9(4cm)	A2	Accept 25(cm) from correct working.
		A1 for $(BC^2 =) 622 \cdot 2(1)$
		Method Radians Gradians
		Evaluating
		cos29 using 47·31 24·472
		29grads
13. 2×10 ⁷²	B1	
14. (Area scale factor =) (855/225) ² OR (225/855) ²	B1	$3.8^2 = 14.44$ OR $(19/5)^2$ OR $361/25$ OR $(5/19)^2$ OR $25/361$ or equivalent.
5300 × (855/225) ² OR 5300 ÷ (225/855) ²	M1	23/301 of equivalent.
$= 76532(cm^2)$	A1	CAO
15. 23·5 – <u>0·725</u>	M2	If many attempts are offered without a
8-35		method/answer being identified then mark final
		attempt. Accept 23-4999 or 8-34999 (or using recurring
		dot notation).
		Do not accept truncated values of 23·49 or 23·499 or 8·349 or 8·3499.
		Award M1 for correct use of values
		$23 < b \le 24$, $0.72 \le c < 0.73$ and $8.3 < d \le 8.4$
		OR award M1 for correct use of 2 of the 3 correct
		limits. Allow sight of 23·413() for M2.
		- , ,
= 23-41	A1	Only award A1 if M2 gained.

16.(a)	B1	Third box
16.(b) Correct sketch of $y = -1/x$ in appropriate 2 quadrants with axes as asymptotes with no extra curves in the other quadrants.	B2	Penalise -1 for the curling away from the asymptotes at the extremities only if B2 previously awarded. If not B2, award B1 for one of the following: Correct sketch in 1 quadrant with axes as asymptotes with no more than 1 incorrect curve in another quadrant Correct sketch in appropriate 2 quadrants with axes as asymptotes with extra incorrect curves in one or two of the other quadrants for two curves sketched appropriately in both quadrants but not clearly with intention of axes as asymptotes Correct sketch of y = +1/x in appropriate 2 quadrants with axes as asymptotes.

17.(a) $\frac{1}{5} \times \frac{2}{7} \times \frac{3}{8}$	M1	
$= \frac{6}{280} \left(\frac{3}{140} \right) \text{ ISW}$	A1	Accept a decimal answer of 0.021(4). Allow an answer of 0.02 from correct working.
17.(b) [1–'all pass to the left'–'all pass to the right'] $1 - \left(\frac{1}{5} \times \frac{2}{7} \times \frac{3}{8}\right) - \left(\frac{4}{5} \times \frac{5}{7} \times \frac{5}{8}\right)$	M2	Award only M1 if further incorrect work seen. FT from part (a) provided <1. M1 for any one of the following: $1 - \left(\frac{1}{5} \times \frac{2}{7} \times \frac{3}{8}\right) \left[= \frac{137}{140} \right]$ $1 - \left(\frac{4}{5} \times \frac{5}{7} \times \frac{5}{8}\right) \left[= \frac{9}{14} \right]$ $\left(\frac{1}{5} \times \frac{2}{7} \times \frac{3}{8}\right) + \left(\frac{4}{5} \times \frac{5}{7} \times \frac{5}{8}\right) \left[= \frac{53}{140} \right]$
$=\frac{174}{280}\left(\frac{87}{140}\right)$ ISW	A1	CAO Accept a decimal answer of 0.62(1)
17.(b) Alternative method 1 ['2 pass left and 1 right'+'2 pass right and 1 left'] $\left(\frac{1}{5} \times \frac{2}{7} \times \frac{5}{8}\right) + \left(\frac{1}{5} \times \frac{5}{7} \times \frac{3}{8}\right) + \left(\frac{4}{5} \times \frac{2}{7} \times \frac{3}{8}\right)$ $+ \left(\frac{4}{5} \times \frac{5}{7} \times \frac{3}{8}\right) + \left(\frac{4}{5} \times \frac{2}{7} \times \frac{5}{8}\right) + \left(\frac{1}{5} \times \frac{5}{7} \times \frac{5}{8}\right)$	M2	Award only M1 if further incorrect work seen. M1 for any one of the following:
$=\frac{174}{280}\left(\frac{87}{140}\right)ISW$	A1	CAO Accept a decimal answer of 0.62(1)
17.(b) Alternative method 2 [G no ball + D no ball + H no ball] P(G no ball) + P(D no ball) + P(H no ball) =	M2	Award only M1 if further incorrect work seen. M1 for any one of the following: $P(G \text{ no ball}) = \left(\frac{1}{5} \times \frac{2}{7} \times \frac{5}{8}\right) + \left(\frac{4}{5} \times \frac{2}{7} \times \frac{5}{8}\right) \text{ or } \left(\frac{2}{7} \times \frac{5}{8}\right) \left[=\frac{5}{28}\right]$ $P(D \text{ no ball}) = \left(\frac{4}{5} \times \frac{2}{7} \times \frac{3}{8}\right) + \left(\frac{4}{5} \times \frac{5}{7} \times \frac{3}{8}\right) \text{ or } \left(\frac{4}{5} \times \frac{3}{8}\right) \left[=\frac{3}{10}\right]$
		P(H no ball)= $\left(\frac{1}{5} \times \frac{5}{7} \times \frac{3}{8}\right) + \left(\frac{1}{5} \times \frac{5}{7} \times \frac{5}{8}\right) \text{ or } \left(\frac{1}{5} \times \frac{5}{7}\right) \left[=\frac{1}{7}\right]$
$=\frac{174}{280}\left(\frac{87}{140}\right)ISW$	A1	CAO Accept a decimal answer of 0.62(1)
17.(b) Alternative method 3 [G receives 2 balls + D receives 2 balls + H receives 2 balls] P(H left, D right)+P(G left, H right)+P(D left, G right) = $ \left(\frac{3}{8} \times \frac{5}{7}\right) + \left(\frac{1}{5} \times \frac{5}{8}\right) + \left(\frac{2}{7} \times \frac{4}{5}\right) $	M2	Award only M1 if further incorrect work seen. M1 for any one of the following: $P(H \ left, \ D \ right) = \left(\frac{3}{8} \times \frac{5}{7}\right) \left[=\frac{15}{56}\right]$ $P(G \ left, \ H \ right) = \left(\frac{1}{5} \times \frac{5}{8}\right) \left[=\frac{1}{8}\right]$ $P(D \ left, \ G \ right) = \left(\frac{2}{7} \times \frac{4}{5}\right) \left[=\frac{8}{35}\right]$
$=\frac{174}{280}\left(\frac{87}{140}\right)ISW$	A1	CAO Accept a decimal answer of 0.62(1)

18.(a) EĈG = 30(°) Sight of CÊG = 45(°) AND CĜE = 105(°)	B1 B2	Angles must be clearly stated or seen on the diagram. B1 for $C\hat{E}G = 45(^{\circ})$ OR $C\hat{G}E = 105(^{\circ})$ Sight of $C\hat{E}G = 45(^{\circ})$ -implies previous B1. FT 'their $E\hat{C}G = 30$ ' to calculate: • $C\hat{G}E = '$ their $30'/2 + 90$ • $C\hat{E}G = 90 - '$ their $30' \times 3/2$ Use of correct angles implies B1B2.
<i>CE</i> = <u>5×sin105</u> sin45	M2	FT 'their $\hat{CGE} = 105$ ' AND 'their $\hat{CEG} = 45$ '. M1 for $\underline{CE} = \underline{5}$ sin105 sin45 Award M2 for a correct use of 'two-step' trigonometric relationship. M0 otherwise.
= $6.8(3\text{cm}) \text{ or } \frac{5+5\sqrt{3}}{2} \text{ (cm)}$	A1	Allow 7(cm) from correct working.
18(a) Alternative version to find OC first and then use triangle OCE		Angles must be clearly stated or seen on the diagram.
CÔE = 30(°) Sight of CÊG = 45(°)	B1 B1	Sight of CÊG = 45(°) implies previous B1 FT 'their EĈG = 30' to calculate: CÊG = 90 – 'their 30' × 3/2
$\left(OC = \frac{2.5}{\sin(15)} = \right)$		
9.6(5cm) or 9.7(cm) or $5\sqrt{6} + 5\sqrt{2}$ (cm)	B1	Or equivalent e.g. using the sine rule in triangle OCG FT 'sin (½ of 'their CÔE = 30')
CE = <u>9·6(5)×sin30</u> sin45	M2	Use of correct side and angles implies B1B1B1. FT 'their derived OC' AND 'their CÔE = 30' AND 'their CÊG = 45'. M1 for CE = 9.6(5) sin30 sin45 Award M2 for a correct use of 'two-step' trigonometric relationship. M0 otherwise.
= $6.8(3cm)$ or $5+5\sqrt{3}$ (cm)	A1	Allow an answer of 7(cm) from correct working.
18.(b) ½×5×6·8(3)×sin30	M1	FT their solution in (a), provided it does not refer to a side other than CE AND 'their EĈG = 30(°)'.
= $8.5(cm^2)$ or $\frac{25+25\sqrt{3}}{8}$ (cm ²)	A1	

19.	$x + 3 + 5x = 7x^2 \text{ OR } x + 3 = x(7x - 5) \text{ OR}$	M1	Clearing the denominator of x .
	$x+3=7x^2-5x$		May be seen in stages, e.g. first writing $\frac{x+3}{x}$ as $1 + \frac{3}{x}$, etc
	$7x^2 - 6x - 3 = 0$	A1	CAO '= 0' required, but may be implied by an attempt to use the quadratic formula or if $a=7,b=-6,\ c=-3$ used in the quadratic formula.
	$\chi = \frac{-(-6)\pm\sqrt{(-6)^2-4\times(7)\times(-3)}}{2\times(7)}$	M1	This substitution into the formula must be seen for M1, otherwise award M0A0A0. FT 'their derived quadratic equation equated to zero' provided of equivalent difficulty (a, b and c must be non-zero). Allow one slip in substitution for M1 only, but must be correct formula. This can be awarded as a single attempt which may be seen anywhere in the solution for solving their quadratic equation equated to zero.
	$=\frac{3\pm\sqrt{30}}{7}\left(\frac{6\pm\sqrt{120}}{14}\right)$	A1	Can be implied from at least one correct value of \boldsymbol{x} evaluated, provided M1 awarded.
	x = 1.21() AND -0.35()	A1	CAO for their quadratic equation. Both solutions required. Accept 1·2 and -0·4.